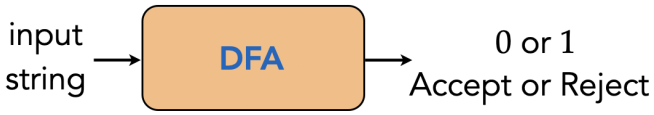
Computetional Model:

Machine = Computer = Program = Algorithm:

Let's assume 2 things about our universe:

Introducing Deterministic Finite Automata (DFA)



Properties:

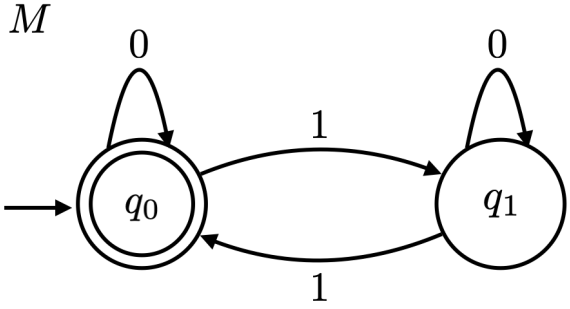
State diagram of a DFA:

DFA M *solves* language L if the following holds:

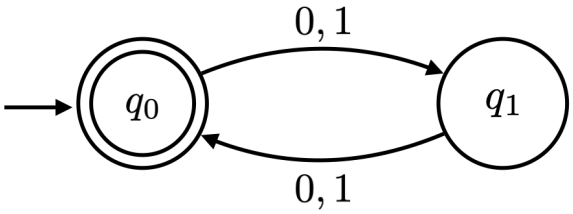
For a DFA M, $L(M) =$

DFA Examples

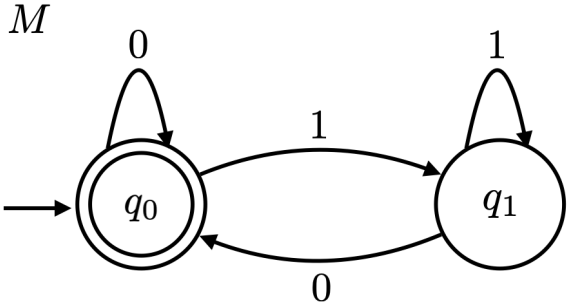
M



M

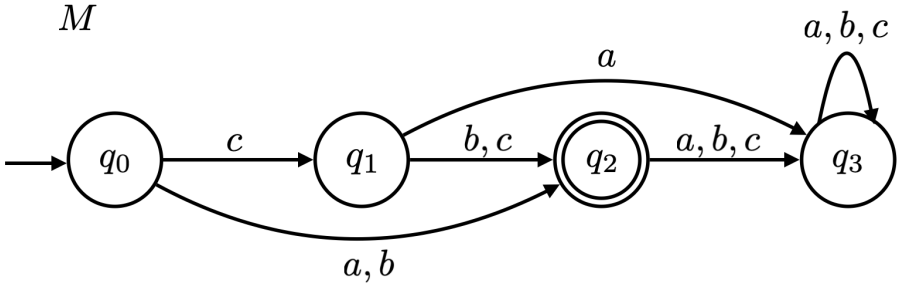


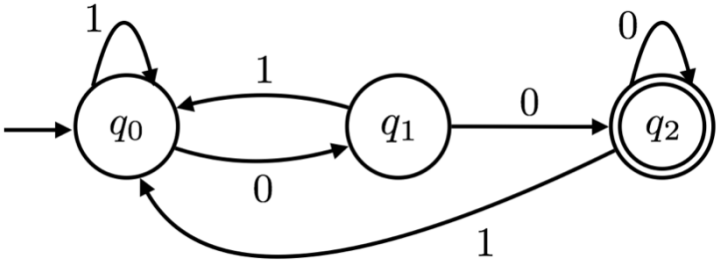
M



$\Sigma = \{a, b, c\}$

M





DFA Construction Practice

$L = \{110, 101\}$

$\{0,1\}^* \backslash \{110, 101\}$

$L = \{x \in \{0,1\}^* : \ x \text{ starts and ends with the same bit}\}$

$L = \{x \in \{0,1\}^* : \ |x| \text{ is divisible by 2 or 3}\}$

$L = \{110, 110110, 110110110, \ldots\}$

$L = \{x \in \{0,1\}^* : \ x \text{ contains the substring 110}\}$

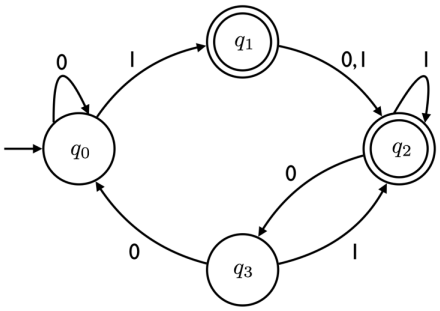
$L = \{x \in \{0,1\}^* : \ 10 \text{ and 01 occur equally often in } x\}$

DFA: A Programming Language

```
def foo(input):
  i = 0;
  STATE 0:
    if (i == input.length): return False;
    letter = input[i];
    i++;
    switch(letter):
      case '0':  go to STATE 0;
      case '1':  go to STATE 1;

  STATE 1:
    if (i == input.length): return True;
    letter = input[i];
    i++;
    switch(letter):
      case '0':  go to STATE 2;
      case '1':  go to STATE 2;
  •••
```

input = | 0 | 1 | 1 | 1 | 1 |



Formal Definitions

Deterministic Finite Automata: A *deterministic finite automata* (DFA) M is a 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F),$$

where

-
-
-
-
-

For $q \in Q$ and $w \in \Sigma^*$, define $\delta^*(q, w) =$

DFA accepting a string:

Regular Language:

Questions:

1. Are all languages regular?
2. Are there other ways to tell if a language is regular (other than constructing a DFA)?

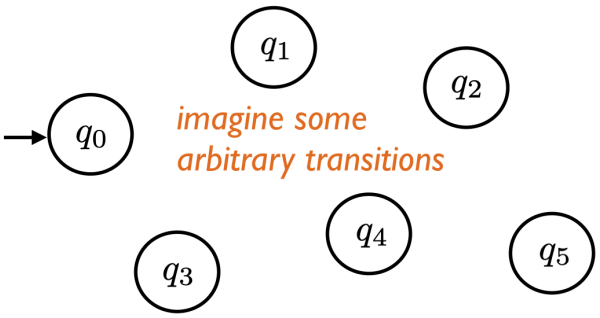
Non-regular Languages

Theorem: The language $L = \{0^n 1^n : n \in \mathbb{N}\}$ is non-regular.

Intuition:

Warm-up: Suppose a DFA with 6 states decides $L = \{0^n 1^n : n \in \mathbb{N}\}$.

Input: 0000000011111111



Proof written formally:

The overall strategy:

1. *Set up a proof by contradiction.*

2. *Pick your pigeons.*

3. *Reach a contradiction.*

Exercise: Show $L = \{c^{251}a^n b^{2n} : n \in \mathbb{N}\} \subseteq \{a, b, c\}^*$ is non-regular.

Exercise: Show $L = \{a^{2^n} : n \in \mathbb{N}\} \subseteq \{a\}^*$ is non-regular.