CS Life Lesson:

Types of Graphs:

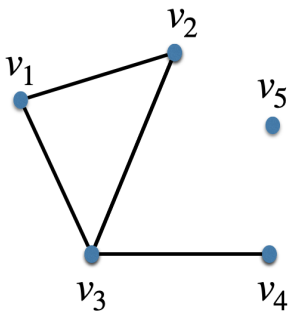
Definition (Simple Undirected Graph):

Example: $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ $E = \{\{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_5, v_6\}\}$
Picture:

Notation: $n =$ $m =$

Suppose $e = \{u, v\}$ is an edge. We say:

Definition (neighborhood):



$N(v_1) =$

$N(v_3) =$

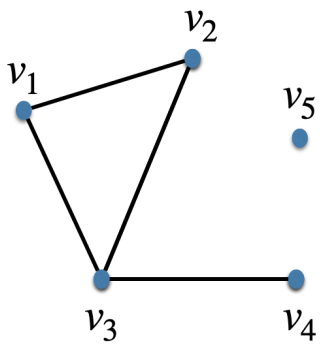
$N(v_5) =$

Definition (degree):

Definition (d-regular graph):

Lemma (Handshake Lemma):

Proof:



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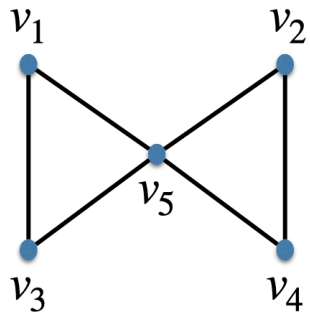
Question: Is it possible to have a graph with 251 vertices in which each vertex is adjacent to exactly 5 other vertices?

Definition (walk):

Definition (path):

Fact:

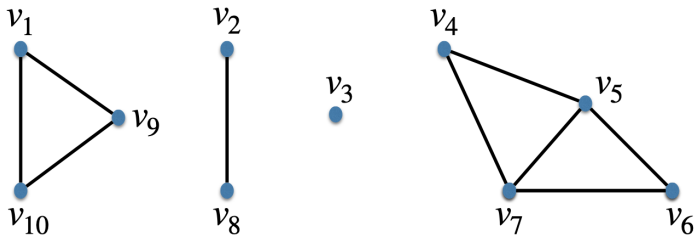
Definition (circuit):



Definition (cycle):

Definition (acyclic graph):

Definition (connected graph):



This graph has 4 **connected components**:

A graph is connected if and only if:

Question: What is the least number of edges needed to connect n vertices?

Theorem:

Proof: Imagine the following process:

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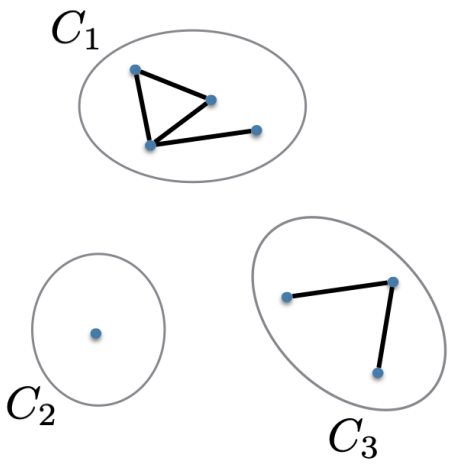
n isolated vertices $\longrightarrow$
n connected components (CCs) $\longrightarrow$

Consider a step of adding an edge back.



2 possibilities:

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Definition (tree):

leaf:
internal node:
rooted tree:

Minimum Spanning Tree (MST) Algorithm

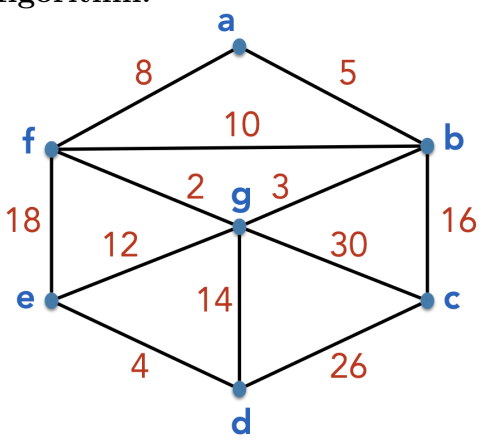
Input:

Output:

Observation:

Convenient assumption:

Algorithm:



Lemma (MST Cut Property):

Proof Idea: Proof by contradiction.

Let T be the MST.