

Introduction

GOAL:

Exponential running time examples

Bounded Entscheidungsproblem

Subset Sum Problem

Traveling Salesperson Problem (TSP)

Satisfiability Problem (SAT)

Sudoku Problem

Scheduling Problem

The P vs NP question is the following:

Identifying and dealing with intractable problems:

(A More Modest) Goal:

Reductions

Let A and B be two languages. Want to define " $A \leq B$ " to mean:

Definition (Polynomial-time Reduction):

The 2 sides of reductions

- 1.
- 2.

Gathering evidence for intractability:

C-hardness and C-completeness

Definition (C-hard):

Definition (C-complete):

Observation:

Good evidence for $A \notin P$:

Main Goal Reduces to:

Complexity Class NP

Super-duper informal:

Informal:

Semi-informal:

Which of the following are in NP?

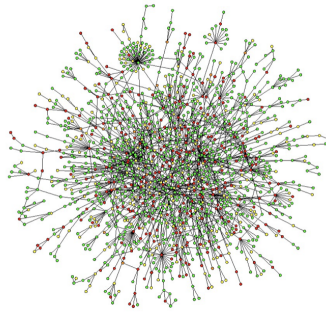
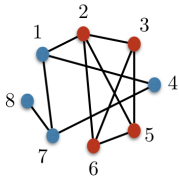
Subset Sum, Traveling Salesperson Problem (TSP), SAT, Circuit-SAT, Sudoku, HALTS, $\{0^k 1^k : k \in \mathbb{N}\}$

Definition (NP):

Example (CLIQUE is in NP):

CLIQUE problem:

Given $\langle G, c \rangle$ where G is a graph and $c \in \mathbb{N}^+$, output True iff G contains a *clique* of size c .



CLIQUE is in NP. Here is the verifier:

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def V(x, u):
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For proof of correctness, need to show:

- 1.
- 2.
- 3.

2 Observations About NP

- 1.
- 2.

NP-completeness

Is there any language that is NP-complete???

Theorem (Cook-Levin 1971-1973):

How do you show a language is NP-complete?

P vs NP Question

If A is NP-hard, that seems to be good evidence that $A \notin P$, if ...

Importance of the P vs NP Question:

Opinions of experts.

	P \neq NP	P = NP	Ind	DC	DK
2002	61(61%)	9(9%)	4(4%)	1(1%)	22(22%)
2012	126 (83%)	12 (9%)	5 (3%)	5 (3%)	1(0.6%)

What does NP stand for anyway?