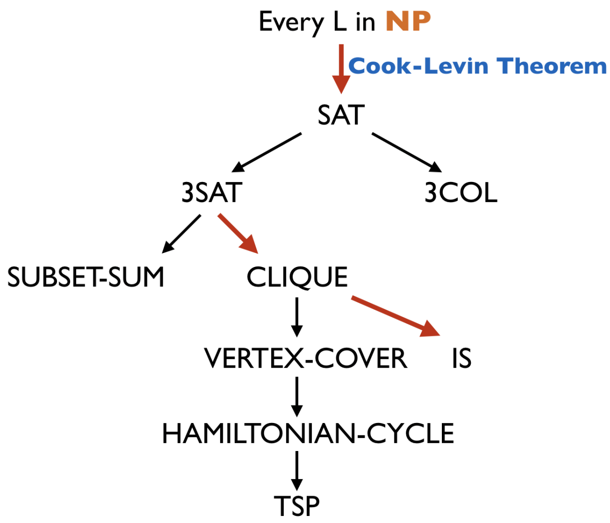


Cook vs Karp Reductions

Cook Reduction:

Karp Reduction:

CLIQUE reduces to IND-SET

Clique: A set S of vertices in a graph $G = (V, E)$ such that for all $u, v \in S$, $\{u, v\} \in E$.

Independent set: A set S of vertices in a graph $G = (V, E)$ such that for all $u, v \in S$, $\{u, v\} \notin E$.

CLIQUE
Input: $\langle G, k \rangle$ where G is a graph and k is a positive int.
Output: Yes iff G contains a clique of size k.

IND-SET
Input: $\langle G, k \rangle$ where G is a graph and k is a positive int.
Output: Yes iff G contains an independent set of size k.

Theorem: CLIQUE Karp reduces to IND-SET

Intuition:

Steps needed to establish a Karp reduction:

1.

2.

3.

Test Your Intuition

kCOL
Input: $\langle G \rangle$ where G is a graph.
Output: Yes iff G is k-colorable.

Which of the following are true?

- 3COL $\leq_m^P$ 2COL is known to be true.
- 3COL $\leq_m^P$ 2COL is known to be false.
- 3COL $\leq_m^P$ 2COL is open.
- 2COL $\leq_m^P$ 3COL is known to be true.
- 2COL $\leq_m^P$ 3COL is known to be false.
- 2COL $\leq_m^P$ 3COL is open.

3SAT reduces to CLIQUE

3SAT
Input: $\langle \varphi \rangle$, where φ is s Boolean formula in "conjunctive normal form" in which every clause has exactly 3 literals.
Output: Yes iff φ is satisfiable.

To show a Karp reduction from 3SAT to CLIQUE, we need to:

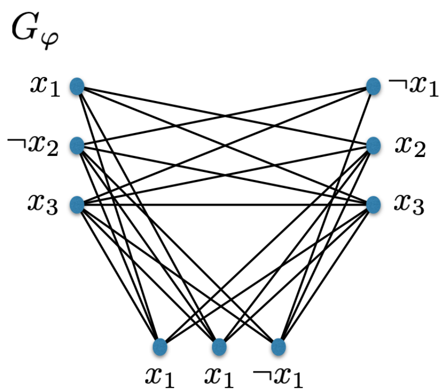
1.
2.
3.

Strategy:

What is a "good" proof that $\langle \varphi \rangle \in$ 3SAT?

1. Construction of the map $f : \Sigma^* \to \Sigma^*$.

$\varphi = (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_1 \vee \neg x_1)$



The construction of G_φ:

2. φ is satisfiable iff G_φ contains an m-clique.

3. Creation of G_φ is poly-time.

Proof of Cook-Levin Theorem (Super High Level)